I.J. Intelligent Systems and Applications, **2022, 4, 21-36**
Published Online August 2022 in MECS (http://www.mecs-press.org/)
DOI:10.5815/ijisa.2022.04.03

Using Rough Set Theory for Reasoning on Vague Ontologies

Mustapha Bourahla
Computer Science Department, University of M'Sila, M'Sila, 28000, Algeria
E-mail: mustapha.bourahla@univ-msila.dz

Received: 24 December 2021; Accepted: 21 March 2022; Published: 08 August 2022

Abstract: Web ontologies can contain vague concepts, which means the knowledge about them is imprecise and then query answering will not possible due to the open world assumption. A concept description can be very exact (crisp concept) or exact (fuzzy concept) if its knowledge is complete, otherwise it is inexact (vague concept) if its knowledge is incomplete. In this paper, we propose a method based on the rough set theory for reasoning on vague ontologies. With this method, the detection of vague concepts will insert into the original ontology new rough vague concepts where their description is defined on approximation spaces to be used by extended Tableau algorithm for automatic reasoning. A prototype of Tableau's extended algorithm is developed and tested on examples where encouraging results are given by this method to demonstrate that unlike other methods, it is possible to answer queries even in the presence of incomplete information.

Index Terms: Vagueness, Rough Sets, Fuzzy Sets, Description Logics, Fuzzy Description Logics, Ontology Web Language, Automatic Reasoning.

1. Introduction

In order to represent knowledge about a real world of individuals (objects), we need to describe properties on concepts and relationships between them. The properties description is done in Description Logics (DLs) [1] by means of ontologies (or knowledge bases). Ontology is a pair of a terminological box and an assertions box, which is defined on a set of concepts, a set of abstract (concrete) roles (relations) and a set of individuals. The terminological box consists of a finite set of axioms on concepts and on roles (as inclusion axioms). However, the assertions box of a knowledge base provides a description of the world itself. Individuals (objects) are introduced via their names, the concepts to which they belong (sets or classes), and their relationships with other individuals.

Ontologies are the definition of domain concepts (extensions) and the relations between them. Ontologies representing knowledge, are expressed in well-defined formal languages for example, Web Ontology Language (OWL2) [2-4], which are based on expressive description logics (as SROIQ(D) for OWL2), [1, 5]. The open world assumption is used in the semantics of DLs, as opposed to the closed world assumption, which assumes that the world is limited to what is expressed. Ontology concepts are language adjectives that correspond to the meaning of object classes (individuals). We will have a difficulty with unclear concepts if the meaning is inadequate (imperfect).

Almost every concept we use in natural language is vague. As a result, whereas vagueness is common in everyday language, it has been avoided in ontology formulations until now. As a result, common sense reasoning based on natural language must rely on vague concepts rather than classical logic. Individuals communicating within a shared linguistic context rely heavily on vague concepts, which are fundamental to natural language. Some underlying internally consistent approach governs the use of vague concepts. We'll concentrate on learning how an intelligent agent can employ vague concepts to convey information and meaning as part of a larger strategy for communication.

If an ontology has at least a vague definition of a concept, we call it vague. If a concept (an extension) defines a meaning gap that prevents us from determining the membership of some objects, it is vague (vague intension). Formalisms for coping with vagueness have begun to play an increasingly important role in Web and Semantic Web research [6-9]. These formalisms have been successfully applied in ontology matching, data integration, multimedia information processing and information retrieval [10]. Furthermore, dealing with vagueness is a crucial aspect of natural language Web interfaces. Ontology languages and logics have been developed to describe areas in which the information to be represented is accompanied with (quantitative) vagueness (or imprecision).

The following example illustrates the issue. Consider an ontology in a domain about cars that defines a concept called *Expensive*. The concept's meaning is vague or imprecise. We can divide the concept *Expensive* into three sub-extensions: a clearly costly extension (certain car prices are clearly expensive), a clearly cheap extension (others are clearly inexpensive cars), and a vagueness extension (average car costs are neither expensive nor cheap). The absence of rigorous knowledge leads to imprecise definition (representation) of concepts, which is the basis of this indecision.

Lotfi Zadeh's concept of a fuzzy set [11] was the first successful approach to vagueness. Fuzzy description logics (FDLs) are approximate rather than exact logics [12] that underpin ways of reasoning [13-15]. The fuzzy knowledge base can be thought of as a collection of assertion constraints. As a result, the inference is seen as a propagation of these constraints. Another approach to vagueness is the rough sets theory, which is a generalization of the fuzzy sets theory [16-19]. We connect two crisp sub-concepts with every vague concept: the first sub-concept consists of all individuals who definitely belong to the concept, and the second sub-concept consists of all individuals who might belong to the concept. A boundary area is made up of individuals who cannot be categorized individually to a concept or its complement using current knowledge. As a result, unlike a crisp concept, any vague concept has a non-empty boundary region.

Both theories take two different approaches to the concept of vagueness. The fuzzy membership in fuzzy theory expresses gradualness of knowledge, whereas the indiscernibility connection in rough set theory expresses granularity of knowledge. In this paper, we present a method for expressing vague concepts in OWL2 based on a combination of rough sets theory and fuzzy sets theory, as well as an extension of the Tableau algorithm for reasoning over vague ontologies. We'll use a rough vague concept to depict a vague concept. A pair of approximations makes up a crude imprecise concept. Its first approximation depicts the objects that might be associated with the concept. The second approximation will represent the objects that possibly belong to its complement.

A rough membership grade interval will be associated with each instance assertion. Its lower bound denotes the smallest rough membership degree, while its upper bound denotes the largest rough membership degree. A truth gap is the difference between the lower and upper bounds (knowledge degree). The knowledge is exact if the truth gap is zero, which is the case with fuzzy concepts. Furthermore, the concept is crisp and we have extremely precise knowledge if both the lower and upper bounds are one (the individual clearly belongs to the concept) or zero (the individual definitely does not belong to the concept).

The growing popularity of description logics and their application, as well as the requirement to deal with vagueness, particularly in the Semantic Web, is drawing many scholars and practitioners' attention to description logics that can deal with vagueness. Any concept instance will have a degree of membership determined by a defined fuzzy function in this situation. Extended fuzzy description logics, which are supported by fuzzy semantics and fuzzy reasoning, are used to deal with vagueness in fuzzy theory. The fuzzy description logics are used in a variety of fields [20].

Many imprecision reasoning tools are developed using fuzzy description logic, but to our knowledge, there are no tools developed on approximate set theory. This prototype developed to reason about imprecision from approximate set theory aims to be more general and to unify all approaches to reasoning about imprecision descriptions.

This paper is organized as follows. Related works are presented in the next section. The third section introduces some basic definitions we rely on by presenting Web Ontology Language (OWL2) and its correspondent description logic (SROIQ(D)). Then a rough set based vagueness method is presented to show how to express vague concepts and to describe the characteristics of vague ontologies. The fifth section presents the extended version of Tableau algorithm, to reason over vague ontologies using this rough set based vagueness method. At the end, we conclude this paper by conclusions and perspectives.

2. Related Works

There are numerous works dealing with vagueness in the literature, and the majority of them represent it as a concept property, such as those based on fuzzy logics. This vagueness method is a generalization of our previous work [21]. We proposed vagueness description with meta-level logic programming to characterize vague ontologies in the paper [7]. These vagueness descriptions are used as inputs to a meta-level vagueness reasoning technique based on the extended tableau algorithm. The extended tableau method is designed to respond to queries even when there is insufficient information [22-24].

The work in [6] presents a tool to support the consensual creation of fuzzy data types using aggregation of specifications to be developed by experts in the domain. This work proposes solutions for the big problem of building Fuzzy OWL2 ontology, which is the definition of fuzzy membership functions for real-valued fuzzy sets that are called fuzzy data types in the Fuzzy OWL2 terminology.

In [14], the authors provide a reasoning algorithm for Fuzzy OWL2 EL for instance/subsumption checking decision problems with a complexity of polynomial time. This reasoning algorithm for fuzzy DLs, which have been proposed as an extension to classical DLs with the aim of dealing with fuzzy concepts under standard and Gödel semantics, has performance similar to the main reasoning algorithm of OWL2 EL.

In [25], the authors suggest a fuzzy expansion of the OWL2 RL language. The OWL2 RL axioms will be translated into equivalent fuzzy Datalog rules when interpreted in a fuzzy context. It is shown that this is not achievable for all axioms in general; however the authors show that this problem can be mitigated to a great amount. They conducted an experiment with a number of well-known ontologies to show that such axioms are rarely employed in practice..

In their paper [26], they offer a rough set based ontology matching method to deal with uncertainty while recognizing similar concepts across different ontologies. This research optimizes concepts for matching by employing concept type categorization as a rough set concept for indiscernibility relations and applying the realism criterion to make decisions under uncertainty. Using the criterion of realism, this integrated approach minimizes the number of concepts examined for matching among uncertain entities compared to existing systems and improves the accuracy of outcomes.

The authors of [27] present a technique for data modeling and user knowledge extraction using a rough set approach, where different rough set algorithms such as K-nearest neighbors (KNN), decision rules (DR), decomposition tree (DT), and local transfer function classifier (LTF-C) are used for an experimental setup. The suggested method's experimental setup is tested by using the dataset available in the UCI web repository, and the methodology has discovered its accuracy for the appropriate use of data modeling and user knowledge. The proposed study's findings show that the model is both useful and efficient, as well as having a high level of accuracy. Different performance indicators, such as F-score, precision, accuracy, recall, specificity, and misclassification rates, are used to assess the validity of the suggested classification algorithms.

The findings of SMS were presented in [28], which were based on publications retrieved from Web of Science and Scopus and automated keyword mapping using a bibliometric analysis tool. The main finding indicates that ontology and fuzzy logic contribute to ISs by expanding traditional IS to become intelligent IS, which can be used to solve complex, fuzzy, and semantically rich (ontological) information collection, saving, processing, sharing, and reasoning in a variety of application domains based on user needs in different countries.

3. Preliminaries on Description Logics and OWL2 Language

Web Ontology Language (OWL2) [1-3] has three sub-languages (logical fragments of OWL2 DL), called tractable profiles [4]. Each profile is designed towards a specific use. The three profiles are OWL2 EL, OWL2 QL and OWL2 RL. The fragment OWL2 EL is based on EL family of description logics (existential and conjunction). It does not allow disjunction and universal restrictions. It is useful for ontologies with large conceptual part. The fragment OWL2 QL (Query Language) is useful for large datasets already stored in relational data bases. It does not allow existential and universal restrictions to a class expression or a data range. Their reasoners can answer complex queries and they can use query rewriting technique. The fragment OWL2 RL (Rule Language) does not allow existential quantification to a class and union and disjoint union to class expressions. These restrictions allow OWL2 RL to be implemented using rule-based technologies such as rule extended data base management systems, Jess, Prolog, etc. It is useful for large datasets stored as RDF triples and it allows fast (polynomial) query answering using rule-extended data bases.

The OWL2 DL alphabet is made up of three sets of names: the atomic concepts that correspond to classes as sets of objects, the atomic roles that correspond to relationships as binary relations on objects, and the individuals (objects). It also includes a collection of constructors for constructing complex concepts and roles from atomic ones. Interpretations are used to express the formal semantics of DLs. An interpretation of SROIQ(D) is represented by a pair $I=(\Delta^I,(.)^I)$ where Δ^I is the domain of I, and $(.)^I$ is the interpretation function to assign for every $A\in\mathcal{C}$ a subset $(A)^I\subseteq\Delta^I$, for every $o\in\mathcal{R}$ a relation $(o)^I\subseteq\Delta^I\times\Delta^I$, (object role), for every $c\in\mathcal{R}$ a relation $(c)^I\subseteq\Delta^I\times\mathcal{D}$, (concrete role), where $\mathcal{D}$ is a data type as integer and string and for every $a\in\mathcal{I}$, an element $(a)^I\in\Delta^I$. The roles (object or concrete) are referred to as properties; if their ranges of values are individuals (individual relationships), they are referred to as object properties. Concrete (data) attributes are those whose range values are concrete data (relation between an individual and a concrete data). The following syntax can be used to express the set of SROIQ(D) complex concepts:

$$\begin{aligned} C ::= \;& T \mid \bot \mid A \mid \{a_1,\cdots,a_n\} \mid \neg C \mid C\cap D \mid C\cup D \mid \exists o.Self \mid \forall o.Self \mid \\ & \exists o.C \mid \forall o.C \mid \exists c.P \mid \forall c.P \mid \geq n\; s.C \mid \leq n\; s.C \end{aligned} \tag{1}$$

Where the symbol T is used for the universal concept, $\bot$ is used for the empty concept, A is used to represent an atomic concept, a for an individual, C and D are used to represent general concepts, o for an object role, c for a concrete role, s for a simple role w.r.t. $\mathcal{R}$, and n for a non-negative integer. The symbol P is used to represent a predicate over a concrete data domain that can have the form:

$$P ::= DataType[\sim value] \mid P\cap P \mid P\cup P, \quad \sim\in\{<,\leq,>,\geq\} \tag{2}$$

Any known data type, such as integer, real, string, and so on, can be used. According to their grammatical structure, the interpretation function is extended to complex concepts and roles. The ability to declare properties of concepts and relationships between them is required to represent real-world domains. In DLs, properties are asserted through the use of an ontology (or knowledge base). A SROIQ(D) ontology is represented by the pair $\mathcal{O} = \langle \mathcal{T}, \mathcal{A} \rangle$, where $\mathcal{T}$ is its terminological box and $\mathcal{A}$ is its assertions box.

A finite set of axioms on concepts and roles make up the terminological box. To build a hierarchy (taxonomy) on the names of concepts and roles, there are inclusion axioms on concepts, objects, and concrete roles, (inclusion axioms are denoted by $C \subseteq D$, $C \equiv D$ is the abbreviation of $C \subseteq D \wedge D \subseteq C$ and the denotation $r_1 \subseteq r_2$ is used for role inclusion, where r_1 and r_2 are object (concrete) roles, the same equivalence abbreviation will be applied on roles). There are axioms for concepts and roles descriptions and roles characteristics. A concept description axiom will be used to assert if a concept is equivalent to, subclass of, or disjoint with a concept expression using the syntax given in (1). By the same way, an axiom for object role description states if it is equivalent to, sub role of, inverse of, disjoint with, or super role of a composition (chain) of object roles. The axioms for concrete roles description are quite different, which are about equivalence, sub property, disjunction, and the ranges are expressions defining union of data type domains as formulated in (2). The characteristic axioms on roles are used to assert if an object role is functional, inverse functional, transitive, symmetric, asymmetric, reflexive, irreflexive, and if a concrete role is functional.

The assertions box is a finite set of assertions stated on individuals: the membership assertions for concepts ($C(a)$ to state that the object (individual) a is member of C), the membership assertions for roles ($o(a,b)$ to state that the objects a and b are related by the object property o and $c(a,d)$ to state that the object a has the data property (concrete role) c with a value equals d). In addition to membership assertions, there are equality and distinctness assertions on individuals ($a = b$ means a is the same as b, $a \neq b$ means a is different from b). We may make some assumptions on how individuals are interpreted. The unique name assumption states that if a and b are two individuals such that $a \neq b$, then $(a)^I \neq (b)^I$. When this assumption holds, equality and distinctness assertions are meaningless.

With the standard name assumption, the unique name assumption holds, and moreover individuals are interpreted in the same way in all interpretations. The interpretation I is considered as a model of the SROIQ(D) ontology $\mathcal{O} = \langle \mathcal{T}, \mathcal{A} \rangle$, all the assertions in $\mathcal{T}$ and $\mathcal{A}$ are satisfied by this interpretation. In addition, it is a model of assertions satisfied by the ontology $\mathcal{O}$. As a result, a knowledge base's assertions box gives a description of the world. Individuals are introduced by their names, the concepts to which they belong, and their relationships with other individuals. The closed world assumption or the open world assumption, are used in the language's semantics. We believe the universe to be confined to what is presented when we use the closed world assumption. In most databases, this is the assumption that is made. In description logics, the assumption of an open universe takes precedence. The way in which inferences are made in description logics is influenced by this open world premise. With the assumption of an open world, the inference becomes more complicated, and it is frequently required to evaluate numerous possible scenarios for the proof. Another significant feature of description logic, as previously discussed, is that it does not assume that names are unique (the standard names). That is, two distinct names do not always imply the existence of two distinct entities in the described reality.

4. Rough Set-based Vagueness Description

To facilitate the understanding of the principle of this method, we present a simple example. We define the concept *Expensive* (for expensive cars) as vague if it has a deficiency of meaning. As a result, the capacity for meaning is the source of vagueness (it has borderline cases). In the following ontology $\mathcal{O} = \langle \mathcal{T}, \mathcal{A} \rangle$, we define a functional abstract role (object property) pr to give different levels of price for different cars belonging to the concept Car, in a domain about car selling. We define the concepts $High$, $HMiddle$, $Middle$, $LMiddle$ and Low for granular levels of high, high-middle, middle, low-middle and low prices, respectively. The terminological box $\mathcal{T}$ is as follows.

$$\mathcal{T} = \left\{ \begin{array}{l} disjoint(High, HMiddle, Middle, LMiddle, Low), \\ functional(pr), Domain(pr) = Car, Range(pr) = Price, \\ Price \equiv High \cup HMiddle \cup Middel \cup LMiddle \cup Low, \\ High \subseteq Expensive, \\ Expensive \subseteq High \cup HMiddle \cup Middle \cup LMiddle, \\ Low \subseteq \neg Expensive, \\ \neg Expensive \subseteq Low \cup LMiddle \cup Middle \cup HMiddle \end{array} \right\} \quad (3)$$

In this knowledge base (ontology), we assume any car having a price in the granular level high (the price is of the concept $High$) is a definitely expensive car and a definitely no-expensive car if it has a price of the concept Low. For the other cases ($HMiddle$, $Middle$, and $LMiddle$ prices), we cannot decide if the price is expensive or not expensive. In this ontology, the concept $Expensive$ and its complement are subsuming two concepts $High$ and Low (they can be any complex concept expressions) and they are subsumed by two other complex concept expressions $High \cup HMiddle \cup Middle \cup LMiddle$ and $Low \cup HMiddle \cup Middle \cup LMiddle$, respectively. The concepts $High$, $HMiddle$, $Middle$, $LMiddle$ and Low are disjoint crisp concepts, which represent granular description of prices.

A world (model) of a knowledge description (ontology) is defined by an interpretation $I = (\Delta^I, (.)^I)$ over a set of individuals $\mathcal{I}$ using description language semantics. Thus, an interpretation is the semantics of a description (ontology) of a world of individuals (objects). A world is a possible world with respect to an ontology $\mathcal{O}$ (denoted by $W(\mathcal{O})$) if its interpretation I satisfies the ontology $\mathcal{O}$ (denoted by $I \models \mathcal{O}$). We denote the set of all the possible worlds that their interpretations make ontology $\mathcal{O}$ true by $\mathcal{W}(\mathcal{O})$. The ontology $\mathcal{O}$ may be inconsistent, in which case $\mathcal{W}(\mathcal{O}) = \varnothing$. A world of individuals is described by a set of assertions of the form $C(a)$ (concept membership assertion), $o(a,b)$ (object role membership assertion) or $c(a,d)$ (concrete role membership assertion), where C is a concept from the set $\mathcal{C}$, a and b are individuals from the set $\mathcal{I}$, o and c are object and concrete roles from the set $\mathcal{R}$, respectively and d is a data from a domain $\mathcal{D}$.

We adopt the unique (standard) name assumption, which states that equality and distinctness assertions between individuals are meaningless. With the standard name, the individuals are interpreted in the same way in all interpretations. Hence, Δ^I contains the set of individuals $\mathcal{I}$, and for each interpretation I, we have $(a)^I = a$ (then, a is called a standard name, which means the interpretation of an individual is its name of identification in the ontology). The possible world (interpretation) $W(\mathcal{O})$ is built by inference rules upon the consistent axioms in $\mathcal{T}$ and the assertions in $\mathcal{A}$. For example, if we have the following set of assertions:

$$\mathcal{A} = \left\{ \begin{array}{l} High(p_0), High(p_1), HMiddle(p_2), HMiddle(p_3), \\ HMiddle(p_4), Middle(p_5), LMiddle(p_6), LMiddle(p_7), \\ Low(p_8), Low(p_9), Car(c), pr(c, p_3), \\ Expensive(p_4), Expensive(p_6), (\neg Expensive)(p_7) \end{array} \right\} \quad (4)$$

The result of inference on this description is a possible world $W(\mathcal{O})$ with respect to the ontology $\mathcal{O}$. A world of individual's interpretation is a possible world with respect to ontology $\mathcal{O}$ if its interpretation satisfies the ontology $\mathcal{O}$. The possible world with respect to this ontology $\mathcal{O}$ is

$$(Car)^I = \{c\},\ (Price)^I = \{p_0, p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9\},\ (High)^I = \{p_0, p_1\},\ (Low)^I = \{p_8, p_9\},$$
$$(HMiddle)^I = \{p_2, p_3, p_4\},\ (Middle)^I = \{p_5\},\ (LMiddle)^I = \{p_6, p_7\},\ (Expensive)^I = \{p_0, p_1, p_4, p_6\},$$
$$(\neg Expensive)^I = \{p_7, p_8, p_9\},\ (pr)^I = \{\langle c, p_3 \rangle\}.$$

The open world assumption in description logics is used to define vague concepts. Thus, for example, this ontology satisfies the assertions $Expensive(p_0)$ and $(\neg Expensive)(c_9)$ but the assertions $Expensive(p_3)$ and $(\neg Expensive)(p_3)$ are both not satisfied. This means, there is a deficiency of meaning between $Expensive$ and $\neg Expensive$. Consequently, the concept $Expensive$ is considered vague. Thus, the satisfaction of a membership assertion to a vague concept depends on the vagueness. The concept $Expensive$ is extensionally vague and it remains intentionally vague in a world of expensive and non-expensive car prices. This indicates that truth-value gaps exist when a vague concept is extensionally (deliberately) absolutely true, definitely false, or true or false.

4.1. Definition

We define a vague concept C (for example, $Expensive$) by a triple $\langle C^{tt}, C^{tf}, C^{ff} \rangle$ with respect to a set of properties (like pr) over a set of individuals of concept S (like $Price$). The first component C^{tt} is the set of individuals that definitely belong to the concept C, the third component C^{ff} is the set of individuals that definitely belong to the complement of the concept C and the second component C^{tf} is the set of individuals that are in the borderline. For example, the vague concept $Expensive$ can be defined by the triple $\langle Expensive^{tt}$, $Expensive^{tf}$, $Expensive^{ff} \rangle$, where their interpretations are as follows: $(Expensive^{tt})^I = \{p_0, p_1, p_4, p_6\}$, $(Expensive^{ff})^I = \{p_7, p_8, p_9\}$, and $(Expensive^{tf})^I = \{p_2, p_3, p_5\}$. We say the price of a car is expensive (for example, $Expensive(p_0)$) if it is high (this is inferred from the terminological description $\mathcal{T}$ in (3)) or it is asserted in $\mathcal{A}$ of (4) as an expensive car price (for example, $Expensive(p_4)$). It is not expensive if it is low (or asserted as not expensive). If the price is high-middle, middle or low-middle we cannot decide if it is expensive or not. For reasoning purpose, we need to propose a method for representing the vague concepts. The representation is based on defining equivalent classes in the terminological box as Low and $High$. A vague concept will be interpreted by approximation sets using the rough set theory [16-18].

4.2. Rough Vague Concepts

Let S be a concept described in ontology $\mathcal{O}$ and $\sim_S \subseteq (S)^I \times (S)^I$ denotes an equivalence relation on $(S)^I \subseteq \Delta^I$ (we recall that $\Delta^I = \mathcal{I}$) from the world $W(\mathcal{O})$, i.e., $\sim_S$ is a reflexive, symmetric and transitive relation. If two individuals $a, b \in (S)^I$ belong to the same equivalence class, i.e., $a \sim_S b$, we say that they are indistinguishable. For example, if $S \equiv Price$, $a \in (Price)^I$ and $b \in (Price)^I$ then

$$a \sim_{Price} b \equiv (\{a,b\} \subseteq (High)^I) \vee (\{a,b\} \subseteq (HMiddle)^I) \vee \\ (\{a,b\} \subseteq (Middle)^I) \vee (\{a,b\} \subseteq (LMiddle)^I) \vee (\{a,b\} \subseteq (Low)^I) \quad (5)$$

We call the pair $\approx_{\sim_S} = ((S)^I, \sim_S)$ an approximation space. Where, the equivalence relation $\sim_S$ will partition $(S)^I$ into disjoint subsets. This defines the quotient set $(S)^I / \sim_S$ which consists of equivalence classes of $\sim_S$. The equivalence class $[\![a]\!]_{\sim_S}$ which contains a plays dual roles. It is a subset of $(S)^I$ if considered in relation to the interpretation domain and an element of $(S)^I / \sim_S$ if considered in relation to the quotient set. If $(C)^I \subseteq (S)^I$ is an interpretation of a vague concept C in the world $W(\mathcal{O})$, it may not be possible to define $(C)^I$ precisely in the approximation space $\approx_{\sim_S} = ((S)^I, \sim_S)$. The lack of knowledge about elements of $(S)^I$ is represented by the indiscernibility relation $\sim_S$. It is possible to characterize $(C)^I$ by a pair of lower and upper approximations with respect to $\sim_S$. This gives us the notion of rough sets. A rough set is interpreted by three ordinary sets: a reference set $(C)^I \subseteq (S)^I$, a lower approximation $\underline{\approx}_{\sim_S}((C)^I) = \{a \in (C)^I \mid [\![a]\!]_{\sim_S} \subseteq (C)^I\}$ and an upper approximation $\overline{\approx}_{\sim_S}((C)^I) = \{a \in (C)^I \mid [\![a]\!]_{\sim_S} \cap (C)^I \neq \emptyset\}$. By definition, $\underline{\approx}_{\sim_S}((C)^I) \subseteq (C)^I \subseteq \overline{\approx}_{\sim_S}((C)^I)$ and $\underline{\approx}_{\sim_S}((C)^I) = \neg \overline{\approx}_{\sim_S}(\neg(C)^I)$. The pair $(\underline{\approx}_{\sim_S}((C)^I), \overline{\approx}_{\sim_S}((C)^I))$ is called a rough set with a reference set $(C)^I$ [1]. We will introduce the notion of rough vague concepts to describe vague concepts.

4.3. Definition

We describe a vague concept C (for example, $Expensive$) by a rough vague concept denoted by C^*, which is defined by the quadruplet $C^* = \langle \underline{C}, \overline{C}, \overline{\neg C}, \underline{\neg C} \rangle$ over an equivalence relation $\sim_S$ defined on a set of individuals from $(S)^I$ (like $(Price)^I$). The new crisp concepts $\underline{C}$ and $\overline{C}$ are the lower and upper approximations of C, and the other crisp concepts $\overline{\neg C}$ and $\underline{\neg C}$ are the upper and lower approximations of the complement of C.

The extension definition of rough vague concepts introduces four new crisp concepts. The intention of rough vague concepts is defined with respect to a current possible world $W(\mathcal{O})$. For example, the interpretation of $Expensive^* = \left\langle \underline{Expensive}, \overline{Expensive}, \underline{\neg Expensive}, \overline{\neg Expensive} \right\rangle$ is $(Expensive^*)^I = \langle \{p_0, p_1\}, \{p_0, p_1, p_2, p_3, p_4, p_6, p_7\}, \{p_6, p_7, p_8, p_9\}, \{p_8, p_9\} \rangle$. Thus, we have defined the interpretation of the new crisp concepts where p_0 and p_1 are definitely expensive prices ($(\underline{Expensive})^I$), p_8 and p_9 are definitely no expensive prices ($(\underline{\neg Expensive})^I$). But the prices in the set $\{p_0, p_1, p_2, p_3, p_4, p_6, p_7\}$ are possibly (approximately) expensive prices (denoted $(\overline{Expensive})^I$) and the prices in the set $\{p_6, p_7, p_8, p_9\}$ are possibly not expensive ($(\overline{\neg Expensive})^I$). There is no (zero) knowledge about the price p_5.

A possible world $W(\mathcal{O})$ can change if we add assertions on individuals. These changes will update the possible world by new beliefs if these additional assertions are satisfied by the ontology due to the presence of vague concepts and the assumption of the open world. However, if the changes will make the new ontology inconsistent then we should first revise the ontology describing the current possible world and then updating the revised ontology (the current possible world) by these additional assertions. Intuitively, updating $\mathcal{O}$ means the possible world may have changed in such a way that an assertion α, is true. In contrast, revising $\mathcal{O}$ means our description of the set of possible worlds must be adjusted to the possibility of α being true. Ontology $\mathcal{O}$ is complete if for any assertion α, $\mathcal{O}$ implies α or $\mathcal{O}$ implies $\neg\alpha$.

The following axioms $\underline{C} \subseteq C \subseteq \bar{C}$, $\underline{(\underline{C})} = \overline{(\underline{C})} = \underline{C}$ and $\overline{(\bar{C})} = \underline{(\bar{C})} = \bar{C}$ are verified. It is clear from definition that $(\bar{C})^I \cup (\overline{\neg C})^I \subseteq (S)^I$. The rough vague concept C^* is a crisp concept if and only if the following condition is verified in every possible world $W(\mathcal{O})$ from the set $\mathcal{W}(\mathcal{O})$: $((\underline{C})^I = (\bar{C})^I) \wedge ((\underline{\neg C})^I = (\overline{\neg C})^I) \wedge ((\bar{C})^I \cup (\overline{\neg C})^I = (S)^I)$, and it is a fuzzy concept if the condition $\forall a \in (S)^I : ([\![a]\!]_{\sim_S} \cap (C)^I) \cup ([\![a]\!]_{\sim_S} \cap (\neg C)^I) = [\![a]\!]_{\sim_S}$ is verified. Thus, the rough vague concepts are generalization of fuzzy concepts, which are in their turn generalization of the crisp concepts. We will make this in evidence by defining the rough vague membership functions.

Let $\Delta^I = \mathcal{I}$ be a set of individuals or objects, where an object is denoted by a. A fuzzy concept interpretation $(C)^I = \{\langle a, \mu_C(a)\rangle \mid a \in (S)^I\}$ in a domain interpretation Δ^I, is characterized by a membership function, μ_C, as follows: $\mu_C : \Delta^I \to [0,1]$. Crisp concept interpretation is a subset of fuzzy concept interpretation in which the membership function is constrained to the extreme points $\{0,1\}$ of $[0,1]$. A characteristic function refers to the membership function of a crisp concept interpretation. There are a number of definitions for interpretation of fuzzy-concept complement, intersection, and union. If A and B are the interpretation sets of two concepts, we choose the standard max-min system proposed by Zadeh [11], in which fuzzy-concept operations are defined component-wise as: $\mu_{\neg A}(a) = 1 - \mu_A(a)$, $\mu_{A\cap B}(a) = min\{\mu_A(a), \mu_B(a)\}$, and $\mu_{A\cup B}(a) = max\{\mu_A(a), \mu_B(a)\}$. The fact that fuzzy-concept operations are truth-functional is an essential characteristic. Membership functions of the complement, intersection, and union of fuzzy sets can be determined just from the membership functions of the fuzzy sets involved. Instead of approximation, rough membership functions can be used to define rough sets [18]. The interpretation of a rough vague concept $C^* = \langle \underline{C}, \bar{C}, \overline{\neg C}, \underline{\neg C} \rangle$ (for example, *Expensive*) over an interpretation $(S)^I$ of an approximation space (for example $(Price)^I$) is characterized by a true rough vague membership function, $\alpha_C : \Delta^I \to [0,1]$, and a false rough vague membership function, $\beta_C : \Delta^I \to [0,1]$.

The rough vague membership functions for concept C are defined with respect to approximation $\approx_{\sim_S}$. Each individual a has grade interval $\mu_C(a) = [\alpha_C(a), 1 - \beta_C(a)] \subseteq [0,1]$, where $\alpha_C(a)$ is a lower bound on the grade of membership of a derived from the evidence for a to belong to the concept C with respect to the approximation $\approx_{\sim_S}$, and $\beta_C(a)$ is a lower bound on the grade of membership of the negation of a derived from the evidence against a (grade of a to belong to the complement of the concept C with respect to the approximation $\approx_{\sim_S}$). The two membership functions are defined as follows.

$$\alpha_C(a) = \frac{|(C)^I \cap [\![a]\!]_{\sim_S}|}{|[\![a]\!]_{\sim_S}|} \text{ and } \beta_C(a) = \frac{|(\neg C)^I \cap [\![a]\!]_{\sim_S}|}{|[\![a]\!]_{\sim_S}|} \tag{6}$$

Where $|(C)^I|$ denotes the cardinality of $(C)^I$. The rough membership function expresses conditional probability that a belongs to $(C)^I$ given $\sim_S$ and can be interpreted as a degree that a belongs to $(C)^I$ in view of information about a expressed by $\sim_S$. This approach bounds the grade of membership of a to a subinterval $[\alpha_C(a), 1-\beta_C(a)]$ of $[0,1]$. In other words, the exact grade of membership $\mu_C(a)$ of a may be unknown, but is bounded by $\alpha_C(a) \le \mu_C(a) \le 1 - \beta_C(a)$, where $\alpha_C(a) + \beta_C(a) \le 1$. For a rough vague concept t C^*, we say that the interval $[\alpha_C(a), 1-\beta_C(a)]$ is the rough vague value to the object a. For example, if $[\alpha_C(a), 1-\beta_C(a)] = [0.4, 0.7]$, then we can see that $\alpha_C(a)=0.4$, $1-\beta_C(a)=0.7$ and $\beta_C(a)=0.3$. It is interpreted as the degree that object a belongs to the vague concept C is 0.4, the degree that object a does not belong to the vague concept C is 0.3. The precision of the knowledge about a is characterized by the difference $(1-\beta_C(a)-\alpha_C(a))$, which equals to 0.3.

If the difference is small, the knowledge about a is relatively precise; if it is large, then we know little. If $(1-\beta_C(a))$ and $\alpha_C(a)$ are equal, then the knowledge about a is exact, and the rough vague concept theory reverts back to fuzzy set theory. In addition, if $(1-\beta_C(a))$ and $\alpha_C(a)$ are both equal to 1 or 0, depending on whether a does or does not belong to C w.r.t its approximation $\approx_{\sim_S}(C)$, the knowledge about a is very exact and the theory reverts back to ordinary (crisp) sets. As a result, any crisp or fuzzy value can be considered a variant of a rough imprecise value.

Consequently, the vague concept interpretation can be extended to include the grade intervals. For example, the interpretation of the rough vague concept $Expensive^*$ can be defined now as $(Expensive^*)^I = \{\langle p_0,[1,1]\rangle, \langle p_1,[1,1]\rangle, \langle p_2,[1/3,1]\rangle, \langle p_3,[1/3,1]\rangle, \langle p_4,[1/3,1]\rangle, \langle p_5,[0,1]\rangle, \langle p_6,[1/2,1/2]\rangle, \langle p_7, [1/2,1/2]\rangle, \langle p_8,[0,0]\rangle, \langle p_9,[0,0]\rangle\}$, which is a set of pairs of individuals and their membership grade intervals. The interpretation of a concept is now vague if it contains an individual a with a grade interval $\mu_C(a)=[l,u]$, where $l \neq u$. The interpretation is fuzzy if for all the individuals $l=u$, and it is crisp interpretation if $l=u=1$ or $l=u=0$.

It is clear that $\alpha_C(a)+\beta_C(a)\leq 1$ and $0\leq \alpha_C(a)\leq 1-\beta_C(a)\leq 1$. In other words, the grade of membership of a is bounded to a subinterval $[\alpha_C(a), 1-\beta_C(a)]$ of $[0,1]$. The intuition for this rough set-based vagueness description is as follows. The ontology axioms are described in a terminological box $\mathcal{T}$ using description logic. A possible world of individuals will be described by a set of assertions that should satisfy the axioms in $\mathcal{T}$. The possible world can change by inserting new assertions. A concept C is a crisp concept if for any individual a belonging to a reference concept S, we have $\mathcal{O}\models C(a)\vee(\neg C)(a)$, this means $\mu_C(a)$ is either $[1,1]$ or $[0,0]$. A concept C is a fuzzy concept if it is not crisp and

$\forall a\in(S)^I, \mathcal{O}\not\models C(a)\vee(\neg C)(a)\Rightarrow \exists b\in[\![a]\!]_{\sim_S}: \mathcal{O}\models(C(a)\wedge(\neg C)(b))\vee(C(b)\wedge(\neg C)(a))$ and $\mu_C(a)=[l,u]$, where $l=u$. A concept C is a rough vague concept if it is not crisp and not fuzzy, this means $\mu_C(a)=[l,u]$, where $l\neq u$ and $1-\alpha_C(a)-\beta_C(a)>0$.

4.4. Ontology Extension with Rough Vague Concepts

We believe that extending the description logics (web ontology languages) by new syntax and semantics is inappropriate. However, for reasoning over ontology $\mathcal{O}$ containing rough vague concepts (as for example $Expensive$), we need to detect by inference these rough vague concepts and insert them into the ontology. The objective of this process of ontology extension (or ontology refinement) is to detect knowledge which can help describing vague concepts as rough vague concepts. This refinement will augment the ontology $\mathcal{O}$ by new axioms as $\{\underline{C}\subseteq C\subseteq\overline{C}, \underline{\neg C}\subseteq\neg C\subseteq\overline{\neg C}\}$ and by new assertions of the form $\underline{C}(a)$, $\overline{C}(a)$, $\overline{\neg C}(a)$, $\underline{\neg C}(a)$, $(C(a))[l,u]$ and $((\neg C)(a))[l,u]$. This refinement process with the extended Tableau reasoning algorithm will be the subject of the next section. In the following, we will define the logical properties on complex rough vague concepts for this vagueness method.

Let S_1 and S_2 be two concepts from the ontology $\mathcal{O}=\langle\mathcal{T},\mathcal{A}\rangle$. Let $\approx_{\sim_{S_1}}=((S_1)^I,\sim_{S_1})$ and $\approx_{\sim_{S_2}}=((S_2)^I,\sim_{S_2})$ be two approximation spaces built on the equivalence relations $\sim_{S_1}$ and $\sim_{S_2}$, respectively with respect to a possible world $W(\mathcal{O})$. Let $C^*=\langle\underline{C},\overline{C},\overline{\neg C},\underline{\neg C}\rangle$ and $D^*=\langle\underline{D},\overline{D},\overline{\neg D},\underline{\neg D}\rangle$ be two rough vague concepts described on the two approximation spaces $\approx_{\sim_{S_1}}$ and $\approx_{\sim_{S_2}}$, respectively. Let $I=(\Delta^I,(.)^I)$ be an interpretation over a set of individuals $\mathcal{I}$. We extend the ontology $\mathcal{O}$ by rough vague concepts and we define their semantics using the interpretation I and the rough vague membership functions, according to their syntactic structure using C^* and D^*.

The predefined concepts like the universal concept T, the empty concept $\perp$, the atomic concepts A and the nominative concepts $\{a_1,a_2,\cdots,a_n\}$ are defined as crisp concepts and then they will not be considered as vague concepts. Their membership interpretations are $\forall a\ \mu_T(a)=[1,1]$, $\mu_\perp(a)=[0,0]$, $a\in(A)^I\Rightarrow\mu_A(a)=[1,1]$, $a\notin(A)^I\Rightarrow\mu_A(a)=[0,0]$ and $a\in(\{a_1,a_2,\cdots,a_n\})^I\Rightarrow\mu_{\{a_1,a_2,\cdots,a_n\}}(a)=[1,1]$, $a\notin(\{a_1,a_2,\cdots,a_n\})^I\Rightarrow\mu_{\{a_1,a_2,\cdots,a_n\}}(a)=[0,0]$.

We should differentiate between empty concept $\perp$ and empty rough vague concept. The former is a crisp concept where the knowledge about the membership is very exact, no individual should belong to the concept $\perp$ and any complex concept should be different from $\bot$ unless the ontology is inconsistent. An empty rough vague concept will be represented by the quadruplet $\langle\bot,\bot,\bot,\bot\rangle$, its true membership function α and false membership function β are both 0, this means we don't have information if the object belongs to the vague concept or not and then, the membership grade interval is $[0,1]$ (this means the information is unknown, there is no individual from the equivalence class which belongs to the concept or to its complement). However, the definition of $\bot$ means that we know very exactly that no object belongs to it. Furthermore, we define empty vague value, or simply empty value, as $[0,0]$.

The definitions of the complex concepts C of the form $\exists o.Self$, $\forall o.Self$, $\exists c.P$ or $\forall c.P$ will not create vague concepts as they are unqualified restrictions, which will create new complex concepts with restrictions on object or concrete roles without referring to qualified concepts (crisp, fuzzy or vague concepts). The membership interpretation is as defined for crisp concepts. However, if the complex concepts C are of the form $\exists o.D$, $\forall o.D$, $\leq n\, s.D$ or $\geq n\, s.D$ then

$$(\exists o.D)^* \;=\; \exists o.(D^*) = \exists o.\langle \underline{D}, \bar{D}, \overline{\neg D}, \underline{\neg D}\rangle = \langle \exists o.\underline{D}, \exists o.\bar{D}, \exists o.\overline{\neg D}, \exists o.\underline{\neg D}\rangle$$

$$(\forall o.D)^* = \forall o.(D^*) = \forall o.\langle \underline{D}, \bar{D}, \overline{\neg D}, \underline{\neg D}\rangle = \langle \forall o.\underline{D}, \forall o.\bar{D}, \forall o.\overline{\neg D}, \forall o.\underline{\neg D}\rangle$$

$$(\leq n\, s.D)^* = (\leq n\, s.(D^*)) = (\leq n\, s.\langle \underline{D}, \bar{D}, \overline{\neg D}, \underline{\neg D}\rangle) = \langle (\leq n\, s.\underline{D}), (\leq n\, s.\bar{D}), (\leq n\, s.\overline{\neg D}), (\leq n\, s.\underline{\neg D}\rangle)$$

$$(\geq n\, s.D)^* = (\geq s.(D^*)) = (\geq n\, s.\langle \underline{D}, \bar{D}, \overline{\neg D}, \underline{\neg D}\rangle) = \langle (\geq n\, s.\underline{D}), (\geq n\, s.\bar{D}), (\geq n\, s.\overline{\neg D}), (\geq n\, s.\underline{\neg D})\rangle$$

Their rough membership interpretations are, $\forall a \in (\Delta)^I$:

$$\mu_{\exists o.D}(a) = \mu_D(b), \text{ where } \alpha_D(b) = max\{\alpha_D(c) \mid \exists (a,c) \in (o)^I \wedge c \in (D)^I\}$$

$$\mu_{\forall o.D}(a) = \mu_D(b), \text{ where } \alpha_D(b) = min\{\alpha_D(c) \mid \forall (a,c) \in (o)^I \Rightarrow c \in (D)^I\}.$$

$$\mu_{(\leq n\, s.D)}(a) = \mu_D(b), \text{ where } \alpha_D(b) = min\{\alpha_D(c) \,||\, \{(a,c) \in (s)^I \wedge c \in (D)^I \,|\leq n\}$$

$$\mu_{(\geq n\, s.D)}(a) = \mu_D(b), \text{ where } \alpha_D(b) = max\{\alpha_D(c) \,||\, \{(a,c) \in (s)^I \wedge c \in (D)^I \,|\geq n\}$$

The complement of a rough vague concept C^* is denoted by $\neg C^*$, where its approximation property is defined by $\underline{\neg C} = \neg \bar{C}$ and $\overline{\neg C} = \neg \underline{C}$. The complement rough vague membership is defined by $\alpha_{\neg C}(a) = \beta_C(a)$, and $1-\beta_{\neg C}(a) = 1-\alpha_C(a)$. The intersection of two rough vague concepts A^* and B^* is a rough vague concept C^*, written as $C^* \equiv A^* \sqcap B^*$, whose true membership and false membership functions are related to those of A^* and B^* by $\alpha_C(a) = min\{\alpha_A(a), \alpha_B(a)\}$, and $1-\beta_C(a) = min\{1-\beta_A(a), 1-\beta_B(a)\} = 1-max\{\beta_A(a), \beta_B(a)\}$. The intersection has the approximation property, $\underline{A \cap B} = \underline{A} \cap \underline{B}$ and $\overline{A \cap B} \subseteq \bar{A} \cap \bar{B}$.

The union of two rough vague concepts A^* and B^* is a rough vague concept C^*, written as $C^* \equiv A^* \cup B^*$, whose true membership and false membership functions are related to those of A^* and B^* by: $\alpha_C(a) = max\{\alpha_A(a), \alpha_B(a)\}$, and $1-\beta_C(a) = max\{1-\beta_A(a), 1-\beta_B(a)\} = 1-min\{\beta_A(a), \beta_B(a)\}$.

The union has the approximation property, $\underline{A \cap B} \supseteq \underline{A} \cap \underline{B}$ and $\overline{A \cap B} = \bar{A} \cap \bar{B}$.

A rough vague concept C^* is contained in another rough vague concept D^* ($C^* \subseteq D^*$) if and only if $\alpha_C(a) \leq \alpha_D(a)$, and $1-\beta_C(a) \leq 1-\beta_D(a)$ ($[\alpha_C(a), 1-\beta_C(a)] \subseteq [\alpha_D(a), 1-\beta_D(a)]$) for all $a \in \Delta^I$. The approximation property is $C^* \subseteq D^* \Rightarrow \underline{C} \subseteq \underline{D} \wedge \bar{C} \subseteq \bar{D}$. Two rough vague concepts C^* and D^* are equivalent, written as $C^* \equiv D^*$, if and only if, $C^* \subseteq D^*$ and $D^* \subseteq C^*$; that is $\alpha_C(a) = \alpha_D(a)$, and $1-\beta_C(a) = 1-\beta_D(a)$.

5. Reasoning over Rough Vague Ontologies

We say that the interpretation I is a possible world (model) of an ontology $\mathcal{O}=\langle\mathcal{T},\mathcal{A}\rangle$ denoted by $I|=\mathcal{O}$ if I satisfies all the axioms in $\mathcal{T}$ and all the assertions in $\mathcal{A}$. The reasoning task is to check concept and role instances and to answer queries over a satisfied ontology [29, 30]. So, the first step is to verify the ontology satisfaction (verify if ontology $\mathcal{O}$ admits at least one model). Checking concept instance is to verify whether an object (individual) a is a member of a concept C in every possible world (model) of $\mathcal{O}$, i.e., check if $\mathcal{O}|=C(a)$. Checking role instance is to verify if a pair (a,b) of individuals is an instance of a role r in every model of $\mathcal{O}$, i.e., check if $\mathcal{O}|=r(a,b)$. Thus, an ontology $\mathcal{O}$ logically implies an assertion α (concept or role instance) written $\mathcal{O}|=\alpha$, if it is satisfied by all models of $\mathcal{O}$.

There are properties that should be verified to check the satisfaction of ontology $\mathcal{O}=\langle\mathcal{T},\mathcal{A}\rangle$. These properties are divided into properties formulated on axioms in the terminological box (to validate the terminological box) and others formulated on assertions in the assertions box (to verify the satisfaction of assertions box). The terminological box properties are the concepts satisfaction which means every definition of a concept C with respect to $\mathcal{T}$, should not assign its interpretation an empty set of objects by any interpretation model ($\mathcal{T}|\neq C\subseteq\perp$). The other properties are the subsumption between two concepts ($\mathcal{T}|=C\subseteq D\Leftrightarrow\mathcal{T}|\neq C\cap\neg D$), the equivalence ($\mathcal{T}|=C\equiv D\Leftrightarrow\mathcal{T}|\neq C\cap\neg D\wedge\mathcal{T}|\neq\neg C\cap D$) and the disjointness ($\mathcal{T}|=C\cap D\subseteq\perp\Leftrightarrow\mathcal{T}|\neq C\cap D$).

Properties for characteristic axioms on roles are also verified as for example, a functionality axiom ($functional(r)$) is logically implied by $\mathcal{T}$ if for every model I of $\mathcal{T}$, we have that $(a,b)\in r$ and $(a,c)\in r$ implies $b=c$ i.e., $\mathcal{T}|=functional(r)$. On other hand, the property that should be verified with the assertions box is the consistence of all its individual assertions (concept and role membership assertions, equality and distinctness assertions on individuals) with respect to the terminological axioms in $\mathcal{T}$. The consistence property states for example, an individual should not belong at the same time to two disjoint concepts. For checking concept and role instances, as well as query response, a fulfilled (validated) ontology can be employed [31]. These satisfaction properties will be extended to account for ontology vagueness. We will use this proposed vagueness technique based on rough set theory coupled with interpretation based on membership grade intervals as specified in the previous section to extend the reasoning Tableau algorithm to deal with the problem of vague ontologies.

A possible world of individuals can change by adding assertions or incorporating new ontologies which will extend its description ontology. Intuitively, updating $\mathcal{O}$ means the possible world may have changed in such a way that α is true. In contrast, revising $\mathcal{O}$ means our description of the set of possible worlds must be adjusted to the possibility of α being true [20]. Typically, two kinds of modifications are distinguished. The first change is that new information is being updated, which is about the current situation, whereas old beliefs are about the past. As a result, the update is the process of replacing old beliefs with new ones in order to account for the change. The revision is the second change, in which both old beliefs and new information apply to the same circumstance. The revision is addition of a belief while maintaining consistency. The new assertion α must be a consequence of the revised ontology. For the purposes of reasoning about action, the usual approach is to represent a particular action as a pair of a precondition and a post-condition. The precondition for the action encodes what the world must be like in order for the action to be executable. The post-condition describes the immediate consequences resulting from the action. The essential difference between revision and update is a temporal one. The revision is a change to our description of a world that has not it-self changed, while the update is the incorporation into our world description of the fact that the world has changed.

If $\mathfrak{O}$ is a non-empty set of ontologies, a partial order $\langle\mathfrak{O},\leq\rangle$ is defined as, if $\mathcal{O}_1$ and $\mathcal{O}_2$ are two ontologies from $\mathfrak{O}$ we write $\mathcal{O}_1\leq\mathcal{O}_2$, if $\mathcal{O}_1$ is less complete than $\mathcal{O}_2$ (or $\mathcal{O}_2$ extends $\mathcal{O}_1$). The relation $\leq$ (extension relation) is based on comparison of rough vague concepts and it is transitive and antisymmetric. This partial order definition has a canonical normal ontology $\mathcal{O}_n$ which is the least complete ontology to be extended by other complete ontologies. The base ontology of $\mathfrak{O}$ corresponds to description of which all other descriptions are extensions. This base ontology constitutes of terminological axioms and eventually some membership assertions. A condition that can be imposed on domain ontology is its completeability, which states that any intermediate ontology can be extended to a complete ontology. If the ontology $\mathcal{O}$ has a rough vague concept C^*, the membership assertions will update the rough vague concepts as ontology extensions.

5.1. Lemma

(Stability property of $\langle \mathfrak{O}, \leq \rangle$). Let α be an assertion, we say $\langle \mathfrak{O}, \leq \rangle$ is stable if:

$$\forall \mathcal{O}_1, \mathcal{O}_2 \in \mathfrak{O}, \mathcal{O}_1 \leq \mathcal{O}_2 : \mathcal{O}_1 \models \alpha \Rightarrow \mathcal{O}_2 \models \alpha \text{ and } \mathcal{O}_2 \not\models \alpha \Rightarrow \mathcal{O}_1 \not\models \alpha \tag{7}$$

Because the whole ontology may not be available to completely remove the vagueness, the most extended ontology must be used. This means that the truth-value is based on the most comprehensive ontology. When utilizing an intelligent agent or inferred assertions, an ontology can be extended to a complete ontology by acquired assertions as part of an ontology evolution process. The user can import assertions from other domain ontologies, RDF databases, or just add them manually. We suggest an extension of reasoning that can take into consideration these notions of the proposed vagueness technique in the next section. The principle of this reasoning method is to use well-defined rules to expand the ontology assertions represented as subsets until no rule can be applied to at least one subset (satisfaction) or contradictions (clashes) appear within all subsets (unsatisfaction). To use the expansion criteria given below, the terminological box should be normalized. It's crucial to start the inference process with formulas that aren't bound by any terminology. This entails removing the terminological box's definitions (equivalence axioms) and subsumptions (inclusion axioms). If there are no cycles in the definitions (which is most of the time), it will happen by simply replacing all of the terms in the formula with their terminology definitions. Obviously, if a formula term lacks a terminological definition, it remains unmodified. We repeat this process until there are no terms in the resulting formula that have a definition in the language.

The method used to reason with the inclusion axioms is to transform any inclusion axiom of the form $C \subseteq D$ to an equivalent inclusion axiom of the form $T \subseteq \neg C \cup D$. Then, for each introduced individual a in the assertions box, add the assertion $(\neg C \cup D)(a)$. At the end of this process of terminological box elimination, we will have only knowledge of assertions that should be normalized by writing negations only before concept identifiers (negative normal form). The Tableau algorithm expands a finite configuration ($T = \{A_1, \cdots, A_n\}$) of assertions that is represented as a set of subsets, each subset is composed of assertions on individuals, using rules defined below until it gets clashes or no rule can be applied in A_i for $1 \leq i \leq n$. We will have a clash in a subset A_i when a contradiction happens in it. There are two types of contradictions: $\perp(a) \in A_i$ ($\mu_{\perp}(a) = [1,1]$), $C(a) \in A_i \wedge (\neg C)(a) \in A_i$ ($\mu_C(a) = [1,1] \wedge \mu_{\neg C}(a) = [1,1]$). If no expansion rule can be applied in A_i we say that A_i is open. This case can also happen if we have a vague concept. As a solution, we have extended this Tableau algorithm by rules to deal with this situation using the rough set based vagueness method. As defined above, a subset A_i is a set of assertions ($A_i = \{C_1(a_1), \cdots, C_k(a_k)\}$), where each C_i is a concept (class) expression defined by the grammar of (1). We will use the following format for defining the expansion rules.

$$\frac{T, A_i[X]}{T, A_i[Y_1], \cdots, A_i[Y_n]} \tag{8}$$

If X is a subset of assertions from A_i in T then, expand T by adding $A_i[Y_1], \cdots,$ $A_i[Y_n]$, formally

$$(A_i \in T \wedge X \subseteq A_i) \Rightarrow T := T \cup \{A_i \cup Y_i\} \cup \cdots \cup \{A_i \cup Y_n\}$$

The following is the extended Tableau algorithm for reasoning over satisfied vague ontologies. We will present the most useful classical rules: the intersection rule ($Rule_{\sqcap}$), the union rule ($Rule_{\sqcup}$), the rule for restriction by existential quantification ($Rule_{\exists}$) and the rule for restriction by universal quantification ($Rule_{\forall}$), in addition to proposed rules for reasoning over vague ontologies which are defined according to the rough set-based vagueness method.

$$Rule_{\cap} : \frac{T, A_i[(C \cap D)(a)]}{T, A_i[C(a), D(a)]} \{C(a), D(a)\} \not\subseteq A_i$$

$$Rule_{\cup} : \frac{T, A_i[(C \cup D)(a)]}{T, A_i[C(a)], A_i[D(a)]} \{C(a), D(a)\} \cap A_i = \varnothing$$

$$Rule_{\exists} : \frac{T, A_i\left[(\exists r.C)(a)\right]}{T, A_i\left[r(a,b), C(b)\right]} r(a,b) \notin A_i \wedge C(b) \notin A_i$$

$$Rule_{\forall} : \frac{T, A_i\left[(\forall r.C)(a), r(a,b)\right]}{T, A_i\left[C(b)\right]} C(b) \notin A_i$$

For reasoning over vague ontologies using the proposed vagueness method, we have added the following rules that should be applied when the application of classical reasoning rules arrived to situation of open world. These rules will be used to detect the ontology vague concepts modulo the query (instance) and then they will generate new crisp concepts composing rough vague concepts with respect to approximation spaces that can exist in the current possible world.

$$Rule_{\underline{\approxeq}} : \frac{T, A_i\left[C(a), \forall b \in \mathcal{I} : (C \cup \neg D)(b)\right]}{T, A_i\left[\underline{C}_D(a)\right]} \underline{C}_D(a) \notin A_i$$

$$Rule_{\underline{\neg\approxeq}} : \frac{T, A_i\left[(\neg C)(a), \forall b \in \mathcal{I} : (\neg C \cup \neg D)(b)\right]}{T, A_i\left[(\underline{\neg C}_D)(a)\right]} (\underline{\neg C}_D)(a) \notin A_i$$

$$Rule_{\overline{\approxeq}} : \frac{T, A_i\left[C(a), \forall b \in \mathcal{I} : (\neg C \cup D)(b)\right]}{T, A_i\left[\overline{C}^D(a)\right]} \overline{C}^D(a) \notin A_i$$

$$Rule_{\overline{\neg\approxeq}} : \frac{T, A_i\left[(\neg C)(a), \forall b \in \mathcal{I} : (C \cup D)(b)\right]}{T, A_i\left[(\overline{\neg C}^D)(a)\right]} (\overline{\neg C}^D)(a) \notin A_i$$

$$Rule_{\overline{\approx}} : \frac{T, A_i\left[D(a), D(b), \overline{C}^D(a)\right]}{T, A_i\left[\overline{C}^D(b)\right]} a \neq b \wedge \overline{C}^D(b) \notin A_i$$

$$Rule_{\overline{\neg\approx}} : \frac{T, A_i\left[D(a), D(b), (\overline{\neg C}^D)(a)\right]}{T, A_i\left[(\overline{\neg C}^D)(b)\right]} a \neq b \wedge \overline{\neg C}^D(b) \notin A_i$$

$$Rule_{\underline{\simeq}} : \frac{T, A_i\left[\forall a \in \mathcal{I} : D(a) \Rightarrow C(a), \overline{C}^D(b)\right]}{T, A_i\left[\underline{C}_D(a)\right]} \underline{C}_D(a) \notin A_i$$

$$Rule_{\underline{\neg\simeq}} : \frac{T, A_i\left[\forall a \in \mathcal{I} : (\neg D(a)) \Rightarrow (\neg C)(a), \overline{\neg C}^D(b)\right]}{T, A_i\left[(\underline{\neg C}_D)(a)\right]} \underline{\neg C}_D(a) \notin A_i$$

The first set of rules ($Rule_{\underline{\approxeq}}$, $Rule_{\underline{\neg\approxeq}}$, $Rule_{\overline{\approxeq}}$ and $Rule_{\overline{\neg\approxeq}}$) will detect the existence of the rough vague concepts and they proceed as follows. If the terminology after normalization contains axioms of the form $T \subseteq C \cup \neg D$ and $T \subseteq \neg C \cup D$ (where the concept C is considered the rough vague concept to be detected) then the all subsets A_i satisfy the assertions $(C \cup \neg D)(a)$ and $(\neg C \cup D)(a)$ for any inserted individual a in the current possible world. Now if a subset A_i contains an assertion of the form $C(a)$ (or $(\neg C)(a)$) then the concept C is really a rough vague concept. The other rules ($Rule_{\underline{\simeq}}$, $Rule_{\underline{\neg\simeq}}$, $Rule_{\overline{\approx}}$ and $Rule_{\overline{\neg\approx}}$) are used to generate the assertions of the form $\underline{C}_D(a)$, $\overline{C}^D(a)$, $\overline{\neg C}^D(a)$ and $\underline{\neg C}_D(a)$ of individuals in the current possible world with respect to the equivalence relation based on the granular atomic concept D of the approximation space.

After generation of possible rough vague concepts with respect to the current possible world, we need to extend the ontology by assertions of the form $(C(a))[l,u]$ that specify the rough membership grade intervals for individuals belonging to the rough vague concept C^* in open subsets A_i. The rule $Rule_{\underline{\mu}}$ checks if a belongs definitely to the concept C then it will insert the assertion $(C(a))[1,1]$ or if (the rule $Rule_{\underline{\neg\mu}}$) a belongs definitely to its complement and then it will insert the assertion $((\neg C)(a))[1,1]$ (i.e. $(C(a))[0,0]$). The rule $Rule_{\underline{\mu}}$ (or $Rule_{\underline{\neg\mu}}$) computes the rough membership grade intervals for individuals belonging to the upper approximations (or to the upper complement approximations) (i.e. the individuals that are in the borderline).

$$Rule_{\underline{\mu}} : \frac{T, A_i\left[C(a), \underline{C}_D(a)\right]}{T, A_i\left[(C(a))[1,1]\right]} (C(a))[1,1] \notin A_i$$

$$Rule_{\underline{\neg\mu}} : \frac{T, A_i\left[(\neg C)(a), \underline{\neg C}_D(a)\right]}{T, A_i\left[((\neg C)(a))[1,1]\right]} ((\neg C)(a))[1,1] \notin A_i$$

$$Rule_{\bar{\mu}} : \frac{T, A_i\begin{bmatrix} C(a), \bar{C}^D(a), n = \#(C(b)), n' = \#(\bar{C}^D(b)), \\ m = \#((\neg C)(b)), m' = \#(\overline{\neg C}^D(b)) \end{bmatrix}}{T, \begin{cases} \text{if } n' \neq 0 \wedge m' \neq 0 \text{ then } A_i\left[(C(a))[n/n', 1-m/m']\right] \\ \text{elseif } n' \neq 0 \wedge m' = 0 \text{ then } A_i\left[(C(a))[n/n', 1]\right] \\ \text{elseif } n' = 0 \wedge m' \neq 0 \text{ then } A_i\left[(C(a))[0, 1-m/m']\right] \\ \text{else } A_i\left[(C(a))[0,1]\right] \end{cases}} \underline{C}_D(a) \notin A_i \wedge a \neq b$$

$$Rule_{\overline{\neg\mu}} : \frac{T, A_i\begin{bmatrix} (\neg C)(a), \overline{\neg C}^D(a), n = \#(C(b)), n' = \#(\bar{C}^D(b)), \\ m = \#((\neg C)(b)), m' = \#(\overline{\neg C}^D(b)) \end{bmatrix}}{T, \begin{cases} \text{if } m' \neq 0 \wedge n' \neq 0 \text{ then } A_i\left[((\neg C)(a))[m/m', 1-n/n']\right] \\ \text{elseif } m' \neq 0 \wedge n' = 0 \text{ then } A_i\left[((\neg C)(a))[m/m', 1]\right] \\ \text{elseif } m' = 0 \wedge n' \neq 0 \text{ then } A_i\left[((\neg C)(a))[0, 1-n/n']\right] \\ \text{else } A_i\left[((\neg C)(a))[0,1]\right] \end{cases}} \underline{\neg C}_D(a) \notin A_i \wedge a \neq b$$

Where $\#(C(b))$ denotes the number of membership assertions of the form $C(b)$ in the subset A_i. The Tableau algorithm as explained before, expands in each iteration i, a finite configuration ($T^i = \left\{A_1^i, \cdots, A_n^i\right\}$) of a set of n subsets, each subset A_j^i for $1 \leq j \leq n$, is composed of assertions on individuals, using the presented Tableau rules until it gets clashes (unsatisfaction) in all the subsets or no rule can be applied in at least one subset. This subset is called open subset and we should check if it contains rough vague concepts. The configuration length depends on ontology description and property being checked.

5.2. Example

We will explain this algorithm extension on a simple example. We want check the following instance on the example ontology $\mathcal{O} = \langle \mathcal{T}, \mathcal{A} \rangle$ of car prices presented in the previous section (the terminological box $\mathcal{T}$ is presented in (3) and the assertions box $\mathcal{A}$ is presented in (4)). The query is an instance checking of if the price of the car c is expensive", formally, $\mathcal{O} \vDash (Car \cap \exists pr.Expensive)(c)$. We want to check the membership of the individual c to the class expensive cars or the price of the car c, which is p_3, is an expensive price, $Expensive(p_3)$. This means that we want to prove $(\neg(Car \cap \exists pr.Expensive))(c)$ is inconsistent with the ontology description ($\mathcal{O} \not\vDash (\neg(Car \cap \exists pr.Expensive))(c)$). After elimination of terminological axioms in $\mathcal{T}$ and normalization (transformation to the negative normal form) as preliminary steps (iteration number 0) before applying Tableau Rules, we have:

$$T^0 = \left\{ A_1^0 = \left\{ \begin{array}{l} High(p_0), High(p_1), HMiddle(p_2), HMiddle(p_3), \\ HMiddle(p_4), Middle(p_5), LMiddle(p_6), \\ LMiddle(p_7), Low(p_8), Low(p_9), \\ Car(c), pr(c, p_3), Expensive(p_4), \\ Expensive(p_6), (\neg Expensive)(p_7), \\ (\neg Car \cup \forall pr.\neg Expensive)(c) \end{array} \right\} \right\} \quad (9)$$

The application of the rule $Rule_{\sqcup}$ (iteration number 1) gives us

$$T^1 = \left\{ \begin{array}{l} A_1^1 = A_1^0 \cup \{(\neg Car)(c)\} \\ A_2^1 = A_1^0 \cup \{(\forall pr.\neg Expensive)(c)\} \end{array} \right\} \quad (10)$$

In the subset A_1^1, we have the membership assertions $Car(c)$ and $(\neg Car)(c)$, which means clash. For the subset A_2^1, we apply the restriction rule $Rule_{\forall}$ and the result of iteration number 2 is

$$T^2 = \left\{ \begin{array}{l} A_1^2 = A_1^1 = \square \\ A_2^2 = A_2^1 \cup \{(\neg Expensive)(p_3)\} \end{array} \right\} \quad (11)$$

It is clear that the subset A_1^2 is open, then we apply the rules $Rule_{\underline{\approxeq}}$, $Rule_{\underline{\neg\approxeq}}$, $Rule_{\approxeq}$, $Rule_{\overline{\neg\approxeq}}$, $Rule_{\underline{\simeq}}$, $Rule_{\underline{\neg\sim}}$, $Rule_{\approx}$ and $Rule_{\overline{\neg\sim}}$ for generating possible assertions on existing rough vague concepts with respect to possible existence of approximation spaces in the subset A_1^2.

$$T^3 = \left\{ \begin{array}{l} A_1^3 = A_1^2 = \square \\ A_2^3 = A_2^2 \cup \left\{ \begin{array}{l} \underline{Expensive}_{High}(p_0), \underline{Expensive}_{High}(p_1), \\ \underline{\neg Expensive}_{Low}(p_8), \underline{\neg Expensive}_{Low}(p_9), \\ \overline{Expensive}^{High}(p_0), \overline{Expensive}^{High}(p_1), \\ \overline{Expensive}^{HMiddle}(p_2), \overline{Expensive}^{HMiddle}(p_3), \\ \overline{Expensive}^{HMiddle}(p_4), \overline{Expensive}^{LMiddle}(p_6), \\ \overline{Expensive}^{LMiddle}(p_7), \overline{\neg Expensive}^{LMiddle}(p_6), \\ \overline{\neg Expensive}^{LMiddle}(p_7), \overline{\neg Expensive}^{Low}(p_8), \\ \overline{\neg Expensive}^{Low}(p_9) \end{array} \right\} \end{array} \right\} \quad (12)$$

At the end, we apply the rules $Rule_{\underline{\mu}}$, $Rule_{\underline{\neg\mu}}$, $Rule_{\overline{\mu}}$, $Rule_{\overline{\neg\mu}}$ for generating the rough membership grade intervals of membership assertions on individuals belonging to rough vague concepts.

$$T^4 = \left\{ \begin{array}{l} A_1^4 = A_1^3 = \square \\ A_2^4 = A_2^3 \cup \left\{ \begin{array}{l} Expensive(p_4)[0, 0.67], Expensive(p_6)[0, 0.5], \\ (\neg Expensive)(p_7)[0, 0.5], (\neg Expensive)(p_3)[0, 0.67] \end{array} \right\} \end{array} \right\} \quad (13)$$

At the end of expansion process (the expansion process terminates if no rule can be applied on all subsets), we have a final configuration T^4 of a set of subsets $\{A_1^4, A_2^4\}$. The instance satisfaction will be decided if all the subsets A_i contain clashes else if no rough vague concept is detected in open subsets, then the instance is not satisfied. The third case is when an open subset contains a rough vague concept. Thus, we have a situation of incomplete knowledge about the instance satisfaction. Consequently, the instance satisfaction will be rough (approximate) and not exact.

The rough grade interval of an instance satisfaction is computed using the rough membership interpretations of complex rough vague concepts, defined as logical properties of the rough set-based vagueness method. For example, we can compute the rough satisfaction interval $[l,u]$ for the instance $(Car \sqcap \exists pr.Expensive)(c)$ (i.e. $((Car \sqcap \exists pr.Expensive)(c))[l,u]$), by

$$\begin{aligned} l &= \alpha_{Car\sqcap\exists pr.Expensive}(c) = min\{\alpha_{Car}(c), \alpha_{\exists pr.Expensive}(c)\} \\ &= \alpha_{\exists pr.Expensive}(c) = \alpha_{Expensive}(p_3) = \beta_{\neg Expensive}(p_3) = 0.33 \\ u &= 1-\beta_{Car\sqcap\exists pr.Expensive}(c) = 1-min\{\beta_{Car}(c), \beta_{\exists pr.Expensive}(c)\} \\ &= 1-\beta_{\exists pr.Expensive}(c) = 1-\beta_{Expensive}(p_3) = 1-\alpha_{\neg Expensive}(p_3) = 1.00 \end{aligned} \quad (14)$$

The results of running the method on this example show its significance, which is without this vagueness method, the car c, which has the price p_3 of a defined granular concept $HMiddle$ cannot be decided by the classical reasoning algorithm, as $Expensive$ or $\neg Expensive$ because the definition of $Expensive$ is vague (the knowledge is not complete or it is not detailed enough). However, the ontology contains a membership assertion indicating that the price p_4 which is in the same equivalence class of p_3, is an expensive price ($Expensive(p_4)$); this information can help the intelligent agent (the reasoner) to decide that the car c is at least 33% and at most 100% an expensive car, which means

$$((Car \cap \exists pr.Expensive)(c))[0.33, 1.00] \quad (15)$$

6. Conclusion

In this paper, we have presented a vagueness method based on rough set theory to deal with the problem of ontologies containing vague concepts. A vague concept is defined by a set of axioms with which we cannot decide for certain membership assertions (instance checking) with respect to a possible world of individuals. The proposed solution is based on inserting detected rough vague concepts and rough vague membership assertions into the assertions box during the reasoning process. The rough vague memberships are defined by grade intervals specifying the lower and upper membership grades. This insertion will be realized on the inferred open current possible world if it contains vague concepts. The generated vague concepts are described as rough vague concepts over detected approximation spaces with respect to equivalence classes. The extended Tableau algorithm with the defined logical properties of this rough set based vagueness theory will be used to decide about instance checking (queries) where the classical reasoning method was previously unable to decide about them.

Ontologies could have extension (evolution), where assertions may be added, intentionally or as result of inferences. This ontology evolution can change the beliefs and reduces the knowledge level from little to more knowledge until it reaches a level of exact knowledge. This vagueness method based on rough set theory is in accordance with this notion and it is used to extend the current reasoning method to take into account these vagueness principles. In this vagueness method, it is not necessary to add syntax and semantics to the logic SROIQ(D) and their corresponding languages as OWL2, for specifying the rough vague concepts and assertions. The only work needed is the extension of the reasoning algorithm to handle the additional rules which will be based on OWL2 (SROIQ(D)) rough vague concepts and assertions. The Tableau algorithm extension will define structures for the rough vague concepts and rough membership intervals. This vagueness reasoning method is very promising to be integrated as a tool to answer queries in the presence of imprecise and incomplete information within the knowledge descriptions.

References

[1] Baader, *F.*, Horrocks, *I.*, *Lutz, C. &* Sattler, U., (2017). An Introduction to Description Logic, *Cambridge University Press, ISBN 978-0-521-69542-8, 1-255*

[2] Hitzler, P., (2021). A review of the semantic web field, Communications of the ACM, Volume 64, Issue 2, 76–83, https://doi.org/10.1145/3397512

[3] Hitzler, P., Krotzsch, M., Parsia, B., Patel-Schneider, P. F., (2009). OWL 2 Web Ontology Language: Primer, S. Rudolph (Eds.).

[4] Krotzsch, M., (2012). OWL 2 profiles: An introduction to light weight ontology languages, in: T. Eiter, T. Krennwallner (Eds.), Proceedings of the 8th Reasoning Web Summer School, Vol. 7487 of Lecture Notes in Computer Science, Springer, 112-183.

[5] Britz, K., Casini, G., Meyer T., Moodley, K., & Sattler, U., Varzinczak I., (2021). Principles of klm-style defeasible description logics, ACM Trans. Comput. Log., 22(1),1-46.

[6] Huitzil, *I.*, Bobillo, *F.*, Gómez-Romero, J., Straccia, U., (2020). Fudge: Fuzzy ontology building with consensuated fuzzy datatypes, Fuzzy Sets Syst, 401*, 91-112.*

[7] Bourahla, M. (2018). Description and reasoning for vague ontologies using logic programming, IET Softw. 12(5)*, 397-409.*

[8] Kern-Isberner, G. & Lukasiewicz, T., *(2017).* Many Facets of Reasoning Under Uncertainty, Inconsistency, Vagueness, and Preferences: A Brief Survey, Künstliche Intell. 31(1), *9-13.*

[9] Lukasiewicz, T. & Straccia, U., (2007). Managing uncertainty and vagueness in description logics for the semantic web, J. Web Sem, 6 (4), 291-308.

[10] Di Noia, T., Mongiello, M., Nocera, F., *Straccia, U., (2019).* A fuzzy ontology-based approach for tool-supported decision making in architectural design, Knowl. Inf. Syst. 58(1), 83-112.

[11] Zadeh, L. A., (1965). Fuzzy sets, Information and Control 8 (3), 338-353.

[12] Zadeh, L. A., (1975). The concept of a linguistic variable and its application to approximate reasoning, Inf. Sci. 8 (3), 199-249.

[13] Bobillo, F. & Straccia, U., (2011). Fuzzy ontology representation using OWL 2, International Journal of Approximate Reasoning 52, 1073-1094.

[14] Bobillo, F.& *Straccia, U., (2018).* Reasoning within Fuzzy OWL 2 EL revisited. Fuzzy Sets Syst, 351, 1-40.

[15] Straccia, U., (2013). Foundations of Fuzzy Logic and Semantic Web Languages, CRC Studies in Informatics Series, Chapman & Hall.

[16] Akama S., Kudo Y., Murai T. (2020) Overview of Rough Set Theory. In: Topics in Rough Set Theory, Intelligent Systems Reference Library, vol 168, Springer.

[17] Pawlak, Z., (1982). Rough sets, International Journal of Parallel Programming, 11(5), 341-356.

[18] Pawlak, Z., Polkowski, L., Skowron, A., (2008). Rough set theory, in: Wiley Encyclopedia of Computer Science and Engineering.

[19] Zhang, Q., Xie, Q., Wang, G., (2016). A survey on rough set theory and its applications, CAAI Transactions on Intelligence Technology, Volume 1, Issue 4, 323-333.

[20] Dubois, D. & Prade, H. (1994). A survey of belief revision and updating rules in various uncertainty models, Int. J. Intell. Syst. 9 (1), 61-100.

[21] *Bourahla, M. (2015).* Reasoning over Vague Concepts, Artificial Intelligence and Soft Computing - 14th International Conference, ICAISC 2015, Zakopane, Poland, June 14-28, 2015, Proceedings, Part II, Lecture Notes in Computer Science, volume 9120, pp. 591-602, Springer.

[22] Anil Kumar K.M, Bhargava S, Apoorva R, Jemal Abawajy, "Detection of False Income Level Claims Using Machine Learning", International Journal of Modern Education and Computer Science(IJMECS), Vol.14, No.1, pp. 65-77, 2022. DOI:10.5815/ijmecs.2022.01.06

[23] Jayalath Ekanayake, "Bug Severity Prediction using Keywords in Imbalanced Learning Environment", International Journal of Information Technology and Computer Science(IJITCS), Vol.13, No.3, pp.53-60, 2021. DOI:10.5815/ijitcs.2021.03.04

[24] Rajesh Chakraborty, Uttam Kumar Mandal, Rabindra Nath Barman, " A Comparative Study of ANN and GEP Model to Predict the Pressure Drop in the Water Transportation System", International Journal of Information Engineering and Electronic Business(IJIEEB), Vol.12, No.5, pp. 47-57, 2020. DOI:10.5815/ijieeb.2020.05.05

[25] *Stoilos*, G., Venetis, T., Stamou, G. (2015). A Fuzzy Extension to the OWL 2 RL Ontology Language, Comput. J., 58(11), 2956-2971.

[26] Krishnamurthy, S., Janardanan, A., & Akoramurthy, B. (2018). Rough Set Based Ontology Matching, International Journal of Rough Sets and Data Analysis (IJRSDA), 5(2), 46-68. http://doi.org/10.4018/IJRSDA.2018040103

[27] Liao, X., Nazir, S., Shen, J., Shen, B., Khan, S., (2021). Rough Set Approach toward Data Modelling and User Knowledge for Extracting Insights, Complexity, vol. 2021, Article ID 7815418, 9 pages. https://doi.org/10.1155/2021/7815418

[28] Kalibatiene, D., Miliauskaite, J. (2021). A Systematic Mapping with Bibliometric Analysis on Information Systems Using Ontology and Fuzzy Logic, Appl. Sci., 11, 3003, https://doi.org/10.3390/app11073003

[29] Cucala, D. T., Grau, B. C. & Horrocks, I., (2021). Pay-as-you-go consequence-based reasoning for the description logic SROIQ, Artificial Intelligence, 298.

[30] Nuradiansyah, A., (2020). Reasoning in Description Logic Ontologies for Privacy Management, *Künstl Intell* 34, 411–415. https://doi.org/10.1007/s13218-020-00681-8

[31] Steigmiller, A. & Glimm, B., (2021). Parallelised ABox Reasoning and Query Answering with Expressive Description Logics, ESWC 2021, 23-39

Authors' Profiles

Mustapha Bourahla was born in Algeria and has Master degree from the University of Montreal (Canada) and PhD degree from the University of Biskra (Algeria) both in the field of computer science.

He has worked for Bell-Northern Research and Bell Canada for four years and now he is a professor in computer science department at the University of M'Sila (Algeria). He has many publications in the fields of formal methods, ontologies and artificial intelligence: LTL transformation modulo positive transitions. *IET Comput. Digit. Tech. 11(6): 205-213 (2017),* Description and reasoning for vague ontologies using logic programming. *IET Softw. 12(5): 397-409 (2018)* and Stability-based Dynamic Bayesian Network method for dynamic data mining. *Eng. Appl. Artif. Intell. 77: 283-310 (2019).*

How to cite this paper: Mustapha Bourahla, "Using Rough Set Theory for Reasoning on Vague Ontologies", International Journal of Intelligent Systems and Applications(IJISA), Vol.14, No.4, pp.21-36, 2022. DOI:10.5815/ijisa.2022.04.03